

SVD in MIMO Systems

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Singular Value
Decomposition

✓ Can be employed as a tool
for MIMO wireless
communication.

$$\begin{array}{l} \nearrow H = U \Sigma V^H \\ \text{MIMO channel} \\ \text{matrix} \end{array}$$
$$U^H U = I$$
$$V^H V = V V^H = I$$

Σ is a Diagonal
matrix of non-negative
singular values.

Consider the MIMO Model:

$$\bar{y} = H \bar{x} + \bar{w}$$

Diagram illustrating the MIMO model equation $\bar{y} = H \bar{x} + \bar{w}$ with annotations:

- $\bar{y}$ is labeled as the **receive vector** (indicated by a red arrow).
- H is labeled as **rx t** (indicated by a red arrow).
- $\bar{x}$ is labeled as **t x l** (indicated by a purple arrow).
- $\bar{w}$ is labeled as **rx l** (indicated by a blue arrow).

A green arrow points from the equation to the substitution: **Substitute $H = U \Sigma V^H$**

$$\bar{y} = H \bar{x} + \bar{w}$$

$$\bar{y} = U \Sigma V^H \bar{x} + \bar{w}$$

At the receiver, multiply by U^H

$$U^H \bar{y} = U^H (U \Sigma V^H \bar{x} + \bar{w})$$

$$\tilde{y} = \underbrace{U^H U}_{\mathbf{I}} \Sigma V^H \bar{x} + \underbrace{U^H \bar{w}}_{\tilde{w}}$$

$$\tilde{y} = \Sigma V^H \bar{x} + \tilde{w}$$

$$\tilde{\mathbf{y}} = \sum V^H \bar{\mathbf{x}} + \tilde{\mathbf{w}}$$

At the transmitter, perform

$$\bar{\mathbf{x}} = \underbrace{V \tilde{\mathbf{x}}}_{\text{Precoding}}$$

$$\tilde{\mathbf{y}} = \sum V^H \underbrace{V}_{\mathbf{I}} \tilde{\mathbf{x}} + \tilde{\mathbf{w}}$$

$$\tilde{\mathbf{y}} = \sum \tilde{\mathbf{x}} + \tilde{\mathbf{w}}$$

$$\underset{t \times 1}{\tilde{\mathbf{y}}} = \sum_{\substack{t \times t \\ \nwarrow}} \underset{t \times 1}{\tilde{\mathbf{x}}} + \underset{t \times 1}{\tilde{\mathbf{w}}}$$

$$\begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \vdots \\ \tilde{y}_t \end{bmatrix} = \begin{bmatrix} \sigma_1 & 0 & \dots & 0 \\ 0 & \sigma_2 & & \\ \vdots & & \ddots & \\ 0 & & & \sigma_t \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \vdots \\ \tilde{x}_t \end{bmatrix} + \begin{bmatrix} \tilde{w}_1 \\ \tilde{w}_2 \\ \vdots \\ \tilde{w}_t \end{bmatrix}$$

Spatial
Multiplexing

$$\begin{aligned} \tilde{y}_1 &= \sigma_1 \tilde{x}_1 + \tilde{w}_1 \\ \tilde{y}_2 &= \sigma_2 \tilde{x}_2 + \tilde{w}_2 \\ &\vdots \\ \tilde{y}_t &= \sigma_t \tilde{x}_t + \tilde{w}_t \end{aligned}$$

} t parallel
wireless
channels.

$$\tilde{y}_i = \sigma_i \tilde{x}_i + \tilde{w}_i$$

Received Symbol on i th channel
 Gain of i th channel
 Transmit symbol for i th channel
 Gaussian var σ^2
 $i = 1, 2, \dots, t$

There are t parallel channels
 $\Rightarrow t$ information symbols can be transmitted in parallel
 $\rightarrow$ Spatial Multiplexing.

Noise on i th channel
 SVD